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Preliminary Results with Lattice Covariant Gauge

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In this poster we present a few preliminary results obtained using our method to fix generic covariant gauges on the lattice. We have computed the gluon propagator and we have found a sensitive dependence on the gauge parameter.

As proposed in [1, 2], the generic covariant gauge can be defined by replacing the Landau gauge condition with the following form:

$$\partial_\mu A^G_\mu(x) = \Lambda(x) \ + \text{(periodic boundary conditions)} \tag{1}$$

where $\Lambda(x)$ belongs to the Lie algebra of the group. The functional proposed in [1] in order to fix non-perturbatively the condition (1) is

$$H[G] \equiv ||\partial_\mu A^G_\mu - \Lambda||^2 = \int d^4x \mathrm{Tr}\left[(\partial_\mu A^G_\mu - \Lambda)(\partial_\nu A^G_\nu - \Lambda)\right] \ . \tag{2}$$

In fact it has the property

$$\frac{\delta H[G]}{\delta g} \propto \left[D_\nu \partial_\nu (\partial_\mu A^G_\mu - \Lambda)\right] \tag{3}$$

with $G(x) = e^{ig^a(x)T^a}$, where T^a are the Gell-Mann matrices. Eq. (3) shows that $H[G]$ is stationary when the eq. (1) is satisfied. Spurious solutions, that can be generated by the minimization, do not seem to influence our numerical results. On the lattice, in the generic covariant gauge eq. (2), the expectation value of a gauge-dependent operator $\mathcal{O}$ is obtained by

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int d\Lambda e^{-\frac{1}{2\alpha}\int d^4x Tr(\Lambda^2)} \int dU \mathcal{O}(U^{G(\alpha)}) e^{-\beta S(U)} \tag{4}$$

where $G(\alpha)$ is the gauge transformation that minimizes the discretized version of the functional (2) and the Λs follow a gaussian distribution of width α. In fact, on the lattice, the correct adjustment to the measure is built into the simulation recipe and there is no need to compute the Faddeev-Popov determinant. Hence the numerical procedure implied by eq. (4) can be described as follows:

- A gauge configuration $\{U\}$ with periodic boundary conditions according to the gauge invariant weight $e^{-S_W(U)}$ is generated;
- For each $\{U\}$ configuration random matrices $\Lambda(x)$ belonging to the group algebra are extracted according to the gaussian weight of eq. (4);
- Given $\Lambda(x)$, a numerical algorithm minimizes a discretization of the functional $H[G]$. That defines the lattice gauge fixing condition ;
- The expectation value of the lattice gauge dependent operator is then given by the average over the configurations:

$$\langle \mathcal{O} \rangle^{Latt} = \frac{1}{N} \sum_{\{conf\}} \mathcal{O}(U^G) \quad . \tag{5}$$

This is the procedure we will use to compute gauge dependent correlation functions in a generic covariant gauge.

Of course, in order to fix the gauge non-perturbatively on the lattice, it is necessary to discretize the gauge fixing functional relevant for the gauge condition required. The freedom in the choice of the lattice definition of A_μ, as discussed in ref. [4], can be used to build discretizations of the minimizing functional which lead to an efficient gauge fixing algorithm. This was the case for the standard algorithm of the Landau lattice gauge fixing. It is possible to take advantage of the freedom to choose the discretization of the gluon field to find a discretization of $H[G]$ ("driven discretization") such that it takes only a local linear dependence upon $G(\bar{x})$. This aim can be reached by choosing the discretization of each H term in order to guarantee the local linear dependence on $G(\bar{x})$ instead of following a particular definition of A_μ. Using this idea $H[G]$ can be discretized in the following compact form

$$H_L(G) = \frac{1}{Va^4g^2} Tr \sum_x J^G(x) J^{G\dagger}(x) \tag{6}$$

where

$$N(x) = -8I + \sum_\nu \left[U_\nu^\dagger(x-\nu) + U_\nu(x) \right] ; \; J(x) = N(x) - iag\Lambda(x) \; . \tag{7}$$

It is easy to see that locally the functional transforms linearly in $G(\bar{x})$ and in the continuum limit it goes to the functional (2). The functional (6) is semidefinite positive and, unlike the Landau case, it is not invariant under global gauge transformations. The functional $H_L[G]$ can be minimized using the same iterative algorithm adopted in the Landau gauge fixing. In order to follow the convergence of the algorithm, two quantities can be monitored as a function of the number of iteration steps: the functional $H_L[G]$ itself and

$$\theta_H = \frac{1}{V} \sum_x Tr[\Delta_H \Delta_H^\dagger], \text{ where } \Delta_H(x) = \left[X_H(x) - X_H^\dagger(x) \right]_{Traceless} ,$$

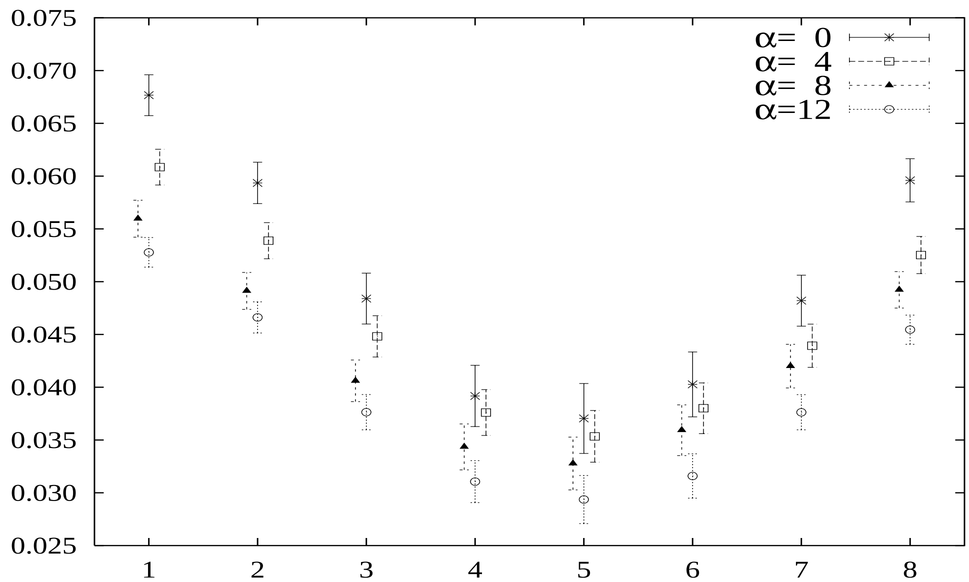

Figure 1: Correlations $\langle \mathcal{A}_i \mathcal{A}_i \rangle(t)$ for different values of α.

and

$$
\begin{aligned}
X_H(x) &= \left[\left(\sum_{\mu} U_\mu(x) J(x+\mu) + U^\dagger_\mu(x-\mu) J(x-\mu) \right) - 8J(x) - 72I \right. \\
&+ \left. 8\,(K(x) + K^\dagger(x)) + N(x) K^\dagger(x) \right] . \qquad (8)
\end{aligned}
$$

Δ_H is invariant under the transformations $\Lambda(x) \to \Lambda(x) + c$ like in the continuum. θ_H decreases (not strictly monotonically) reaching zero when $H_L[G]$ gets constant and it signals that the algorithm has converged. In this feasibility study we have generated 50, SU(3) thermalized configurations using the Wilson action with periodic boundary conditions at $\beta = 6.0$ for a volume of 8^4. Then for each value of α we have extracted a set of $\Lambda(x)$ with a gaussian distribution of width α. We have monitored $H_L[G]$ and θ_H step by step and the chosen quality is $\theta_H \leq 10^{-10}$. Once the configuration have been gauge rotated we have computed the following two point correlation function

$$
\langle \mathcal{A}_i \mathcal{A}_i \rangle(t) \equiv \frac{1}{3V^2} \sum_{i=1,3} \sum_{\mathbf{x},\mathbf{y}} Tr \langle A_i(\mathbf{x}, t) A_i(\mathbf{y}, 0) \rangle. \qquad (9)
$$

In eq. (9) the standard definition is used for the gluon field. Our result for the correlator $\langle \mathcal{A}_i \mathcal{A}_i \rangle(t)$ (9), relevant for the investigation of the QCD gluon sector, are shown in Fig. 1. The relative statistical errors are comparable with the Landau gauge case with the same number of configurations. Even with a small volume and a small number of configurations, the gauge dependence of the gluon

propagator is clearly shown. The α dependence of the gluon propagator shown in Fig. 1 does not seem to be re-absorbed by an overall scaling factor.

This plot shows the feasibility of our procedure to study the gauge dependence of physically interesting correlators.

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